

Finite Size Effects in Quark-Gluon Plasma Formation

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Using lattice simulations of quenched QCD we estimate the finite size effects present when a gluon plasma equilibrates in a slab geometry, *i.e.*, finite width but large transverse dimensions. Significant differences are observed in the free energy density for the slab when compared with bulk behavior. A small shift in the critical temperature is also seen. The free energy required to liberate heavy quarks relative to bulk is measured using Polyakov loops; the additional free energy required is on the order of $30 - 40 \text{ MeV}$ at $2 - 3 T_c$.

Lattice gauge theory has given us considerable information about bulk thermodynamics, but finite size effects have up to now been studied using simplified, phenomenological models. We study the behavior of a gluon plasma restricted to a slab geometry, with the longitudinal width much smaller than the transverse directions. This inner region is heated to temperatures above the bulk deconfinement temperature, surrounded by an outer region which is kept at a temperature below the deconfinement temperature, providing confining boundary conditions for the inner region. Details of this work are given in [1].

By allowing the coupling constant $\beta = 6/g^2$ to vary, a spatially dependent temperature can be introduced. We have chosen the temperature interface to be sharp, in such a way that the lattice is divided into two spatial regions, one hotter and one colder. The quenched approximation simplifies the role of the cold region, because below T_c , the dominant excitation at low energies is the scalar glueball. Temperatures near T_c are smaller than glueball masses by about a factor of four, so glueballs play no the pressure in the hadronic phase is essentially zero. We thus expect the slab thermodynamics to be largely insensitive to the precise temperature of the region outside the slab, as long as it is sufficiently low. In full QCD, this insensitivity to the outer temperature would not hold, due to pions. Note that the role of boundary conditions here is quite different from

that in bubble nucleation. In that case, both β_{in} and β_{out} are taken to be near T_c . [2][3]

The slab is given a fixed lattice width, $w = 6a$, rather than of fixed physical width. Since $N_t = 4$, wT is fixed at $3/2$. At higher temperatures, the width in physical units is somewhat smaller than the longitudinal size of the plasma formation region expected in heavy ion collisions. While the use of equilibrium statistical mechanics to study gluon plasma properties during the early stages of plasma formation may appear suspect, a simple estimate using the Bjorken model [4] shows that when a coin-shaped region of width 1 fermi has expanded to 1.5 fermi, the variation in temperature is only from $0.8T_0$ at the center of the coin to T_0 at its edges.

The free energy density f for the slab was obtained using the standard method [5] of integrating the lattice action with respect to β . We use a convenient convention for the sign of f that is opposite the usual one. In the bulk case, f is then identical to the pressure p .

$$\frac{f}{T^4}|_{\beta_{out}}^{\beta} = N_t^4 \int_{\beta_{out}}^{\beta} d\beta' [\langle S \rangle_T - \langle S \rangle_0] \quad (1)$$

where $S = (1/N) \text{ReTr } U_p$. As in the bulk case, it is necessary to subtract the zero-temperature expectation value from the finite temperature expectation value, in this case using the same pair of β values. In general, quantum field theories with boundaries develop divergences that are not present in infinite volume or with periodic bound-

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any conditions. It is expected that any counterterms would be gauge-invariant and have dimension ≤ 3 ; there are no such counterterms.[6]

In order to take advantage of the data on bulk thermodynamics provided by the Bielefeld group [7], we worked consistently with lattices of overall size $16^3 \times 4$. The values used for each subtraction come from 16^4 lattices with identical values of β_{in} and β_{out} . The value of β_{out} was held fixed at 5.6 while β_{in} varied from 5.6 to 6.3. For comparison, the bulk transition for $N_t = 4$ occurs at $\beta_c(N_t = 4, N_s = 16) = 5.6908(2)$ $\beta_c(N_t = 4, N_s = \infty) = 5.6925(5)$. [7]

Figure 1 shows the free energy density f/T^4 versus T/T_c compared with the bulk pressure. The free energy in the slab is lower than the bulk value by almost a factor of two at $2T_c$. It appears that the slab value is slowly approaching the bulk value, but other behaviors are also possible. Calculations of the finite-temperature contribution to the Casimir effect for a free Bose field contained between two plates show that f/T^4 has a non-trivial dependence on the dimensionless combination wT . [8] [9] It is natural to ask if the corrections to the free energy seen here can be accounted for by the conventional Casimir effect. A straightforward calculation of the free energy of a non-interacting gluon gas confined to a slab shows an increase in the free energy density over the bulk value by a factor of about 1.63 at $wT = 3/2$. The Casimir effect alone is thus unable to explain the reduction of the free energy observed in our simulations. We are currently investigating more elaborate theoretical models which include the effect of a non-trivial Polyakov loop.

A consistency check was performed on the surface effects.[10] The free energy density was calculated for a system at $\beta_{in} = 6.0$ by performing simulations with β_{in} fixed at 6.0 and β_{out} varying from 5.6 to 6.0. Combining these results with the bulk data of reference [7] creates a path equivalent to varying β_{in} while holding β_{out} fixed. For $\beta_{in} = 6.0$ and $\beta_{out} = 5.6$, this gives $f/T^4 = 0.65 \pm 0.04$, to be compared with $f/T^4 = 0.69 \pm 0.03$ for the direct calculation.

The major source of systematic error lies with the choice of boundary conditions for the slab, here set by β_{out} . We have estimated the system-

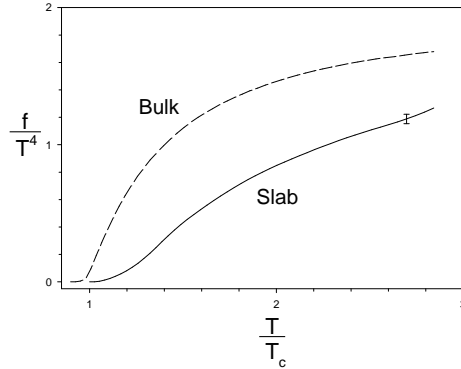


Figure 1. Slab free energy compared with bulk pressure.

atic error associated with the choice of boundary conditions by performing simulations at $\beta_{in} = 6.2$ and $\beta_{out} = 5.5$ on $16^3 \times 4$ and 16^4 lattices. These results suggest that lowering β_{out} from 5.6 to 5.5 reduces the free energy by roughly 10 percent at $\beta_{in} = 6.2$.

We define an effective surface tension $\alpha(w, T)$ by

$$f = p - 2\alpha(w, T)/w \quad (2)$$

where the notation $\alpha(w, T)$ recognizes that the surface tension α does depend on the width of the slab and the internal and external temperatures. In the limits where T approaches T_c and w goes to infinity, this quantity approaches α_0 . Figure 2 shows $\alpha(w, T)/T^3$ versus T/T_c for $wT = 3/2$; representative error bars are shown. Away from the bulk critical point, α/T^3 rises quickly to a peak at about $1.4T_c$, and then falls slowly as T increases. A large non-equilibrium surface tension has also been observed in measurements of the equilibrium surface tension α_0 , where these effects were obstacles to obtaining α_0 . [10]

The multiplicative divergence in the Polyakov loop can be eliminated when comparing bulk expectation values to those in finite geometries. We define

$$\Delta F_Q(\vec{x}) = -T \ln [P_{slab}(\vec{x})/P_{bulk}] \quad (3)$$

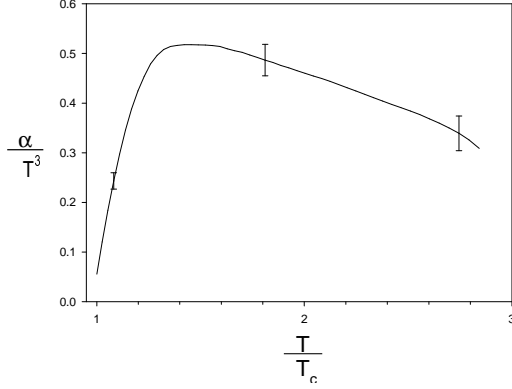


Figure 2. Non-equilibrium surface tension versus T .

as the excess free energy required to liberate a heavy quark in the slab geometry relative to bulk quark matter at the same temperature.

In figure 3, we show the expectation value for the Polyakov loop versus z measured in lattice units for several values of β . Each curve is normalized by dividing the values of the Polyakov loop by the bulk expectation value at the corresponding value of β . Error bars are shown only for even values of z . It is clear that a significant change occurs between $\beta = 5.8$ ($T = 1.23 T_c$) and $\beta = 5.85$ ($T = 1.36 T_c$). For larger values of β , ΔF_Q diminishes to a value of approximately 30–40 MeV in the middle of the slab.

There are significant deviations in the slab geometry from bulk behavior and the ideal gas law, arising from a strong non-equilibrium surface-tension. This non-equilibrium surface-tension can be an order of magnitude greater than the equilibrium value. Surface tension effects also produce a mild elevation of the apparent critical temperature. Measurement of Polyakov loop expectation values relative to bulk shows that the suppression of heavy quark production due to the slab geometry is small.

There are good reasons to call our result an estimate rather than a calculation. Although lattice gauge theory simulations of bulk behavior can be made arbitrarily accurate in principle, in this

case there is uncertainty in the exterior boundary conditions appropriate; the applicability of equilibrium thermodynamics at this early stage of quark-gluon plasma formation can also be questioned. However, this result is the best estimate available now, and further refinements are possible.

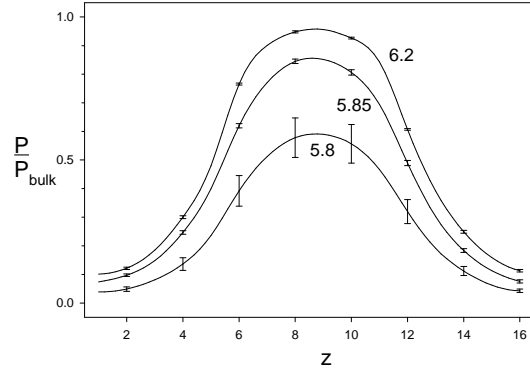


Figure 3. Polyakov loop versus z .

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